Analysis II (42001/52001)
Home Work 11, due Wednesday, April 19
Instructor: Prof. Artem Zvavitch

Problem 1. *If $\sum a_n$ is conditionally convergent, give an argument to show that there exists a rearrangement of this series whose partial sums diverge to ∞.*

Problem 2. *Find an explicit expression for the partial sums of $\sum_{n=2}^{\infty} \ln(1 - \frac{n^2}{)}$ to show that this series converges to $-\ln 2$. Is this convergence absolute?*

Problem 3. *If $\sum a_n$ an is absolutely convergent and $\{b_n\}$ is a bounded sequence, show that $\sum a_n b_n$ is absolutely convergent.*

Now, give an example to show that if the convergence of $\sum a_n$ is conditional and $\{b_n\}$ is a bounded sequence, then $\sum a_n b_n$ may be divergent.

Problem 4. *Decide (with explanation!) which of the following series convergent and which divergent:*

- $\sum\limits_{k=1}^{\infty} \frac{k^3}{(k+1)^{\log k}}$.
- $\sum\limits_{k=1}^{\infty} \left(\frac{3+(-1)^k}{5}\right)^k$.

Problem 5. *Decide for which values $p \in \mathbb{R}$ (with explanation!) the following series convergent and which divergent:*

- $\sum\limits_{k=1}^{\infty} \frac{1}{\log^p k}$.
- $\sum\limits_{k=1}^{\infty} \frac{2^{kp} k!}{k^k}$.

Problem 6. *Suppose that $a_k \geq 0$ and that $a_k^{1/k} \to a$ as $k \to \infty$. Prove that $\sum\limits_{k=1}^{\infty} a_k x^k$ converges absolutely for all $|x| < 1/a$ if $a \neq 0$ and for all $x \in \mathbb{R}$ if $a = 0$.*