Analysis II (42001/52001)
Home Work 12, due Wednesday, April 26
Instructor: Prof. Artem Zvavitch

Problem 1. *Consider a sequence* $\{a_n\}$ *such that* $\lim_{n\to\infty}(n^2 a_n)$ *exists. Show that* $\sum a_n$ *is absolutely convergent.*

Problem 2. *Consider* $a > 0$. *Show that the series* $\sum \frac{1}{1+a^n}$ *is divergent if* $0 < a \leq 1$ *and is convergent if* $a > 1$.

Problem 3. *Check the convergence of following series:*

- $\sum \frac{n!}{n^n}$.
- $\sum \frac{\sqrt{n+1}-\sqrt{n}}{n}$.
- $\sum (\ln n)^{-\ln\ln n}$.

Problem 4. *Prove that the following series converges*

- $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{3^k}$.
- $\sum_{k=1}^{\infty} \frac{\sin kx}{k^p}$, $x \in \mathbb{R}$, $p > 0$.