Introduction to Analysis II
Home Work 3, due Wednesday, February 8.
Instructor: Prof. Artem Zvavitch

Problem 1. *Find*

- $\lim_{x\to\infty} \frac{xe^x}{e^x+\ln x}$
- $\lim_{x\to 0} \frac{e^x \sin x}{x^2+1}$
- $\lim_{x\to 0+} x^{2\sin x}$
- $\lim_{x\to\infty} x^{1/x}$
- $\lim_{x\to 0+} (\frac{1}{x} - \frac{1}{\arctan x})$

Problem 2. *Prove that*

$$1 - \frac{x^2}{2} \le cos x \le 1 - \frac{x^2}{2} + \frac{x^4}{24}.$$

Problem 3. *Show that the sum of two convex functions is again a convex function. Is the same statement true for product? Construct a convex function defined on $\mathbb{R}$ which is increasing for all $x \in \mathbb{R}$*

Problem 4. *Compute e correct to 6 decimal place.*

Problem 5. (Extra 10 points) *Please construct a function which is infinitely differentiable on $\mathbb{R}$ (i.e. $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and $n \in \mathbb{N}$), such that $f^{(n)}(0) = 0$ for all n, but $f(x) \neq 0$ for all $x \neq 0$. Explain why this example do not contradict Teylor's theorem.*