

## Analysis II

Home Work 4, due Wednesday, February 15.

Instructor: Prof. Artem Zvavitch

**Problem 1.** Assume that  $f(x) : I \rightarrow J$  and  $g(x) : J \rightarrow \mathbb{R}$  are convex functions and  $I, J$  are open intervals in  $\mathbb{R}$ . Show that if  $g$  is increasing function then  $g(f(x))$  is a convex function on  $I$ . Would this statement be true without assumption on  $g$  to be increasing?

**Problem 2.** Assume  $f(-1) = f(0) = f(1) = 0$  and  $f$  is a convex function on  $\mathbb{R}$ . Please, find  $f(.5)$ . Can you say anything about  $f(7)$ ?

**Problem 3.** Use the definition of the Riemann integral to show that the function  $f(x) : [0, 2] \rightarrow \mathbb{R}$  such that  $f(x) = 3x$  for  $x \in [0, 1]$  and 7 otherwise is Riemann integrable on  $[0, 2]$ .

**Problem 4.** Consider Riemann integrable function  $f$  on  $[a, b]$  such that  $|f(x)| < M$  for all  $x \in [a, b]$  show that then

$$\left| \int_a^b f dx \right| \leq M(b - a).$$