

**Functions of Real Variables 1 (62051/72051)**  
**Home Work 3, due on Wednesday, SEPTEMBER 25.**  
**Instructor: Prof. Artem Zvavitch.**

**Problem 1.** Give an example of a set  $A$  such that the boundary  $A$  has positive Lebesgue measure but  $m(A) = 0$ .

**Problem 2.** Let  $A$  be the subset of  $[0, 1]$  which consists of all numbers which do not have the digit 4 appearing in their decimal expansion. Find  $m(A)$ .

**Problem 3.** Suppose  $A \subset E \subset B$ , where  $A, B$  are measurable sets with  $m(A) = m(B)$ . Prove that  $E$  is measurable.

WAIT TILL MONDAY

**Problem 4.** Show that a closed set is a  $G_\delta$  and open set is  $F_\sigma$ .

**Problem 5.** Consider a centered, unit euclidean ball  $B = \{x \in \mathbb{R}^d : |x| < 1\}$  let  $v_d = m(B)$  (note you are not required to know or to compute this constant!). Consider  $y \in \mathbb{R}^d$  and  $r > 0$ , let  $B_r(y) = y + rB$  (i.e. a euclidean ball of radius  $r$  centered at  $y$ ). Prove that  $m(B_r(y)) = r^d v_d$ .

**Problem 6.** Let  $\Lambda$  be a diagonal matrix, with all  $\lambda_{i,i} > 0$ . Consider a measurable  $E \subset \mathbb{R}^d$ . Define

$$\Lambda E = \{\Lambda x : x \in E\}.$$

Prove that  $\Lambda E$  is measurable and  $m(\Lambda E) = [\det(\Lambda)]m(E)$ .