

Functions of Real Variables 1 (62051/72051)
Home Work 10, due on Thursday December 5.
Instructor: Prof. Artem Zvavitch.

Problem 1. Consider a rectifiable curve $\gamma(t) : [a, b] \rightarrow \mathbb{R}^2$. Show that there is a tangent line to γ at almost all points of γ . Also, please, make this statement precise.

Problem 2. Let $\gamma(t) : [a, b] \rightarrow \mathbb{R}^2$ be a curve, and suppose it satisfies a Lipschitz condition with exponent $\alpha \in [1/2, 1]$, i.e. there exists a constant $A > 0$

$$|\gamma(t_1) - \gamma(t_2)| \leq A|t_1 - t_2|^\alpha \text{ for all } t_1, t_2 \in [a, b].$$

Prove that $m(\gamma^\delta) \leq C\delta^{2-1/\alpha}$ for all $\delta \in (0, 1]$.

Problem 3. Let F be the Cantor-Lebesgue function discussed in class. Consider the curve $\gamma(t) = (t, F(t))$, $t \in [0, 1]$. Prove that the length $L(0, x)$ (i.e. the length of $\gamma(t)$ for $t \in [0, x]$) is equal to $x + F(x)$. Hence the total length of the curve is 2.