Functions of Real Variables 2 (62051/72051)
Home Work 1, due on Wednesday , January 22.
Instructor: Prof. Artem Zvavitch./ Peter Gordon

Problem 1. *Assume F is of bounded variation and continuous. Prove that $F = F_1 - F_2$, where both F_1 and F_2 are monotonic and continuous.*

Problem 2. *Show that if F is of bounded variation in $[a, b]$, then*

- $\int_a^b |F'(x)|dx \leq T_F(a, b)$.
- $\int_a^b |F'(x)|dx = T_F(a, b)$ *iff F is absolutely continuous.*

Use the las result to show that the formula $L = \int_a^b |z'(t)|dt$ for the length of a rectifiable curve parametrized by $z(t)$ holds iff z is absolutely continuous.

Problem 3. *Let $f : \mathbb{R} \to \mathbb{R}$. Prove that f satisfies the Lipschitz condition*

$$|f(x) - f(y)| \leq M|x - y|,$$

for some M and all $x, y \in \mathbb{R}$, iff f satisfies the following two properties:

- *f is absolutely continuous.*
- *$|f'(x)| \leq M$.*