Functions of Real Variables 1 (62051/72051)
Home Work 3, due on Wednesday, SEPTEMBER 20.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Give an example of a set A such that the boundary A has positive Lebesque measure but $m(A) = 0$.*

Problem 2. *Let A be the subset of $[0, 1]$ which consists of all numbers which do not have the digit 4 appearing in their decimal expansion. Find $m(A)$.*

Problem 3. *Suppose $A \subset E \subset B$, where A, B are measurable sets with $m(A) = m(B)$. Prove that E is measurable.*

WAIT TILL MONDAY

Problem 4. *Show that a closed set is a G_δ and open set is F_σ.*

Problem 5. *Consider a centered, unit euclidean ball $B = \{x \in \mathbb{R}^d : |x| < 1\}$ let $v_d = m(B)$ (note you are not required to know or to compute this constant!). Consider $y \in \mathbb{R}^d$ and $r > 0$, let $B_r(y) = y + rB$ (i.e. a euclidean ball of radius r centered at y). Prove that $m(B_r(y)) = r^d v_d$.*

Problem 6. *Let Λ be a diagonal matrix, with all $\lambda_{i,i} > 0$. Consider a measurable $E \subset \mathbb{R}^d$. Define*

$$\Lambda E = \{\Lambda x : x \in E\}.$$

Prove that ΛE is measurable and $m(\Lambda E) = [\det(\Lambda)]m(E)$.