Functions of Real Variables 1 (62051/72051)
Home Work 4, due on Wednesday October 9 .
Instructor: Prof. Artem Zvavitch.

Problem 1. *Let $F : \mathbb{R} \to \mathbb{R}$ be any function (not necessarily measurable!). Prove that the set of points $x \in \mathbb{R}$ such that*

$$F(y) \leq F(x) \leq F(z)$$

for all $y \leq x$ and $z \geq x$ is a Borel set.

Problem 2. *Show that there is no function $f : \mathbb{R} \to \mathbb{R}$ such that*

- *f is continuous.*
- *$f = \chi_{[0,1]}$ a.e. on $\mathbb{R}$.*

PLEASE, WAIT TILL MONDAY

Problem 3. *Prove that if f is integrable on $\mathbb{R}^d$ and $\delta > 0$, then $f(\delta x)$ converges to $f(x)$ in the L^1-norm as $\delta \to 1$.*