Theory of Matrices
Home Work 11, due Thursday, April 20.
Instructor: Prof. Artem Zvavitch

Problem 1. *Show that if v is an eigenvector of a square matrix A belonging to the eigenvalue λ, then v is also an eigenvector of A^2 belonging to the eigenvalue λ^2.*

Problem 2. *Let A be an invertible matrix. Show that if v is an eigenvector of A belonging to the eigenvalue λ, then v is also an eigenvector of A^{-1} belonging to the eigenvalue $1/\lambda$.*

Problem 3. *Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find the characteristic polynomial of A, the eigenvalues and the eigenvectors of A.*

Problem 4. *Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 4 & -1 \\ -1 & 2 & 1 \end{bmatrix}$. Find the characteristic polynomial of A, the eigenvalues and eigenvectors of A.*

Problem 5. *Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 4 & -1 \\ -1 & 1 & 2 \end{bmatrix}$. Find the bases for the eigenspaces of A. Next, find and compare the algebraic and geometric multiplicities of the eigenvalues. Finally, find a matrix P such that $P^{-1}AP$ is diagonal.*