Theory of Matrices
Home Work 12, due Thursday, April 29.
Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$. *Find the characteristic polynomial* $\Delta_A(t)$ *of* A *and evaluate* $\Delta_A(A)$ *to verify that the Caley-Hamilton Theorem holds for* A*:* $\Delta_A(t) = 0f$ *or any* $n \times n$ *matrix* A

Problem 2. *Show that an* $n \times n$ *matrix* A *is invertible if and only if 0 is not an eigenvalue of* A*.*

Problem 3. *Let* $A = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$. *Find a matrix* P *such that* $A = PDP^{-1}$ *for a diagonal matrix* D*. Next, use this expression to compute* A^{10}*.*

Problem 4. *For matrix* A *from the previous problem find a matrix* B *such that* $A = B^2$*. For this, use the expression* $A = PDP^{-1}$ *with matrices* P *and* D *found in previous problem. Hint: If* $D = C^2$ *for some matrix* C *show that* $A = (PCP^{-1})^2$*.*

Problem 5. *Find a linear substitution which would diagonalize* $q(x, y, z) = 5x^2 + 3y^2 + 12xz$*.*

Problem 6. *Show that if* A *is a square matrix such that* $A^2 = A$*, then the minimal polynomial of* A *is* $m(t) = t$*,* $m(t) = t - 1$*, or* $m(t) = t(t - 1)$*.*