Theory of Matrices
Home Work 1, due Thursday, January 26.
Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $W = \{(a, b, c) \in \mathbb{R}^3 : a + 3b + 7c = 0\}$. *Show that* W *is a vector space over* $\mathbb{R}$ *under the usual vector addition and scalar multiplication. Make sure you explain why* W *is closed under these operations and then check that all the axioms hold true.*

Problem 2. *Let* W *be the set of polynomials* $p(t)$ *of degree* 3 *or less such that* $p(0) = p(1)$. *Show that* W *is a vector space over* $\mathbb{R}$ *under the usual vector addition and scalar multiplication. Make sure you explain why* W *is closed under these operations and then check that all the axioms hold true.*

Problem 3. *Use the vector space axioms to prove the cancellation law: If* V *is a vector space and* u, v, w *are vectors in* V *that satisfy* $u + w = v + w$ *then* $u = v$. *At each step, state which axiom you used.*

Problem 4. *Let* V *be a vector space over a field* K. *Show that for any* $\alpha \in K$ *and* $v \in V$ *we have*

(a) $(-\alpha)v = -(\alpha v)$,
(b) $\alpha(-v) = -(\alpha v)$.

State which axioms or results proved in class you are using to justify each step.

Problem 5. *Check if it is possible to write polynomial* $p(t) = 3t^2 + 2t + 1$ *as a linear combination of polynomials* $p_1(t) = t^2 + t + 1$, $p_2(t) = t + 1$ *and* $p_3(t) = 1$.