Theory of Matrices
Home Work 2, due Thursday, February 2.
Instructor: Prof. Artem Zvavitch

Reminder: $M_n(\mathbb{R})$ - vector space of n by n matrices over $\mathbb{R}$.

Problem 1. *Decide if the each of the following is a subspace of $\mathbb{R}^3$. Justify your answers. (This means provide proofs.)*

(a) $W = \{(a,b,c) \in \mathbb{R}^3 \mid a+b-c=7\}$;
(b) $W = \{(a,b,c) \in \mathbb{R}^3 \mid 2a+3b-6c=0\}$;
(c) $W = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2-z^2=0\}$.

Problem 2. *Determine if each of the following is a subspace of $M_2(\mathbb{R})$. Justify your answers. (This means provide proofs.)*

(a) $W = \{A \in M_2(\mathbb{R}) \mid \det(A) = 0\}$;
(b) $W = \{A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a,b,c \in \mathbb{R}\}$.

Problem 3. *Show (=prove) that the set $W = \{A \in M_n(\mathbb{R}) \mid A^T = -A\}$ is a subspace of $M_n(\mathbb{R})$.*

Problem 4. *Let $A \in M_n(\mathbb{R})$ and let $\lambda \in \mathbb{R}$. Show that $W = \{v \in \mathbb{R}^n \mid Av = \lambda v\}$ is a subspace of $\mathbb{R}^n$.*

Problem 5. *Check if the matrix $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ is in the span of*

$$S = \left\{ \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix} \right\}.$$

Problem 6. *Write the polynomial $f(t) = at^2 + bt + c$ as a linear combination of the polynomials $p_1(t) = (t+1)^2$, $p_2(t) = (t+1)$, and $p_3(t) = 1$.*

Problem 7. *Let*

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ 0 & -1 & 1 \\ -1 & -2 & 0 \end{bmatrix}.$$

Give conditions on the constants a,b,c,d that ensure that the vector $v = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ is in the column span of A. (Hint: For this, you would need to understand if the system $Ax = v$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ has a solution. Find the echelon form of the augmented matrix $[A \mid v]$.)

Bonus 1. *Give an example of a vector space V and two subspaces U and W of V such that $U \cup W$ is not a subspace of V. Now, suppose that U and W are subspaces of the vector space V such that $U \cup W$ is also a subspace of V. Show that this implies that either $U \subset W$ or $W \subset U$.*