Theory of Matrices
Home Work 3, due Thursday, February 9.
Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $S = \{u, v\}$ *be a linearly independent subset of a vector space* V*. Show that* $\{u+v, u-v\}$ *is also a linearly independent subset of* V*. Also show that* $\{u+v, u-v, 6u+3v\}$ *is not linearly independent.*

Problem 2. *Determine if the following set of vectors in* $\mathbb{R}^4$ *is linearly independent:*

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 7 \\ -1 \end{bmatrix} \right\}.$$

Problem 3. *Determine if the following subset of* $\mathrm{P}_3(t)$ *is linearly independent.*

$$S = \{t^3 - 3t^2 + t + 1,\ t^2 + 2t - 1,\ t^3 + t + 1\ , t - 1,\ t^2 + t + 5\}.$$

Problem 4. *Let* $S = \{u_1, u_1, u_3, u_4\}$ *be a linearly independent subset of a vector space* V*. Show that* $\operatorname{span}(u_1, u_2) \cap \operatorname{span}(u_3, u_4) = \{\overline{0}\}$*. (Hint: Let* v *be a vector in the intersection. Show that* v *must be equal to* $\overline{0}$*.)*

Problem 5. *Find a subset of the set*

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\} \subset \mathbb{R}^3$$

that forms a basis for $\operatorname{span}(S)$*.*

Problem 6. *Let* $U = \{(a, b, c, d) \in \mathbb{R}^4 \mid a + b + c - d = 0\}$ *and* $V = \{(a, b, c, d) \in \mathbb{R}^4 \mid a = c,\ b = c + d\}$*. Find a basis and dimension for (a)* U*; (b)* V*; (c)* $U \cap V$*.*

Problem 7. *Find a basis and dimension for the space spanned in* $\mathrm{P}_3(t)$ *by the polynomials* $u = t^3 - 2t + 1$*,* $v = 2t^3 + t^2 - 1$*,* $w = t^3 + 2t^2 + 6t - 5$*.*

Problem 8. *Find a basis for (i) the row space and (ii) the column space of the matrix*

$$M = \begin{bmatrix} -1 & 2 & 1 & -1 & 2 \\ 1 & 1 & -2 & 1 & 1 \\ 2 & -1 & -1 & 1 & 3 \\ 2 & 2 & 1 & 1 & 1 \end{bmatrix}.$$