Theory of Matrices
Home Work 4, due Thursday, February 16.
Instructor: Prof. Artem Zvavitch

Problem 1. *Check that the set* $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ *is a basis for* $M_2(\mathbb{R})$. *Find the coordinate vectors* $[A]_S$ *and* $[B]_S$ *relative to this basis* S *for* $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ *and* $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *where* a, b, c, d *are real numbers.*

Consider a vector space V with a basis $S = \{u_1, \dots, u_n\}$ we say $[v]_S = (v_1, \dots, v_n)$ are coordinates of vector v in basis S if

$$v = v_1 u_1 + \cdots + v_n u_n.$$

Problem 2. *Check that the set* $S = \{t^3 + t^2, t^2 - t, t^2 + t + 1, t^3 + 1\}$ *is a basis for* $P_3(t)$.

Find the coordinate vectors $[v]_S$ *and* $[u]_S$ *relative to this basis* S *for* $u = 2t^3 - t^2 + t - 1$ *and for* $v = at^3 + bt^2 + ct + d$, *where* a, b, c, d *are real numbers.*

WAIT TILL TUESDAY TO DO THIS

Problem 3. *Let* U *and* W *be subspaces of a vector space* V *such that* $V = U \oplus W$. *Show that* $\dim V = \dim U + \dim W$.

Problem 4. *Let* U *and* W *be three-dimensional subspaces of* $\mathbb{R}^5$. *Show that* $U \cap W \neq \{0\}$.