

Theory of Matrices
Home Work 5, due Thursday, February 23.
Instructor: Prof. Artem Zvavitch

Problem 1. Let U and W be subspaces of a vector space V such that $\dim U = 3$, $\dim W = 5$, $\dim V = 7$. Find the possible dimensions of $U \cap W$.

Problem 2. Let U , V , and W be the following subspaces of \mathbb{R}^3 :

$$U = \{(a, b, c) \mid a = b\}, \quad V = \{(a, b, c) \mid a + b + c = 0\}, \quad W = \{(0, b, 0)\}.$$

Show that (a) $\mathbb{R}^3 = U + V$, (b) $\mathbb{R}^3 = V + W$, (c) $\mathbb{R}^3 = U + W$. Which of these sums are direct?

Problem 3. (BONUS 5 pts) Please, check if

$$U = \{p(t) \in P_2 : \text{the equation } p(t) = 0 \text{ has at least one real solution}\}$$

is a subspace of P_2 (of polynomials of degree less or equal to two).

Problem 4. (BONUS 15 pts) Prove that $V = \{p(t) \in P_2 : p(1) = 0\}$ is a subspace of P_2 find a basis and the dimension of V .

Problem 5. (BONUS 15 pts) Let v_1, v_2, v_3 , be vectors in a vector space V . Given that $S = \{v_1, v_2, v_3\}$ are linearly independent, find a basis and the dimension of $\text{span}\{v_1 + 2v_2, v_1 + 2v_3, v_1 + v_2 + v_3\}$ Write your argument in complete sentences.