Theory of Matrices
Home Work 6, due Thursday, March 2.
Instructor: Prof. Artem Zvavitch

Problem 1. *Let* $u = (1, 1, 1, 1)$ *and* $v = (1, 1, 1, 0) \in \mathbb{R}^4$ *Compute the following, with respect to the standard inner product (dot product) on* $\mathbb{R}^4$*:* $\langle u, v\rangle$, $||u||$, $||v||$, *and the exact value of the angle between* u *and* v*.*

Problem 2. *Let* $u = (x_1, y_1, z_1)$ *and* $v = (x_2, y_2, z_2)$ *be vectors in* $\mathbb{R}^3$*. Show that* $\langle u, v\rangle = ax_1x_2+by_1y_2+cz_1z_2$ *define an inner product on* $\mathbb{R}^3$ *if and only if* $a, b, c > 0$*.*

Problem 3. *Let* $u = (x_1, y_1, z_1)$ *and* $v = (x_2, y_2, z_2)$ *be vectors in* $\mathbb{R}^3$*. Show that* $\langle u, v\rangle = x_1y_2z_1 + x_2y_1z_2$ *does not define an inner product on* $\mathbb{R}^3$*.*

Problem 4. *Consider* $P_2(t)$ *with the inner product defined by* $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$*. Find a basis of the subspace* W *orthogonal to* $h(t) = t + 1$*.*

Problem 5. *Find a basis for the subspace* W *of* $\mathbb{R}^5$*, which is the orthogonal complement to the set* $\{u, v\}$*, where* $u = (1, 2, 4, 3, 1), v = (1, 0, 1, 2, 1)$*.*