Theory of Matrices
Home Work 7, due Thursday, March 9.
Instructor: Prof. Artem Zvavitch

Problem 1. *Find an orthonormal basis for the subspace* W *of* $\mathbb{R}^4$*, which is the orthogonal complement to the set* $\{u, v\}$*, where* $u = (1, 0, 3, 1), v = (1, 1, -1, 2)$.

Problem 2. *Let* $w = (-1, 1, 1, 2)$ *be a vector in* $\mathbb{R}^4$*. Find an orthonormal basis for* $w^{\perp}$.

Problem 3. *Show that vectors* $u = (1, -1, 2)$ *and* $v = (1, 3, 1)$ *are orthogonal vectors in* $\mathbb{R}^3$*. Expand* $\{u, v\}$ *to an orthogonal basis of* $\mathbb{R}^3$*. Next, find the coordinates of an arbitrary vector* $(a, b, c) \in \mathbb{R}^3$ *relative to this basis.*

Problem 4. *Given that* $\{u_1, \ldots, u_r\}$ *is an orthogonal set of vectors in* V*, show that* $\{\alpha_1 u_1, \ldots, \alpha_r u_r\}$ *is an orthogonal set for any choice of none zero scalars* $\alpha_1, \ldots, \alpha_r$.

Problem 5. *Find the projection*

(a) *of the vector* $(1, 2, 3)$ *along the vector* $(-1, 2, 1)$ *in* $\mathbb{R}^3$*;*

(b) *of* $f(t) = t^2 + 1$ *along* $g(t) = t - 1$ *in* $P_2(t)$ *with inner product* $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$.

(c) *of* $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ *along* $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ *with inner product* $< A, B >= \mathrm{tr}(B^T A)$.

Recall that trace of a matrix A*, denoted by* $\mathrm{tr}(A)$*, is the sum of its diagonal elements, that is,* $\mathrm{tr} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d$*. Also,* A^T *is the transpose of* A*, that is,* $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$.