Theory of Matrices
Home Work 9, due Thursday, March 23.
Instructor: Prof. Artem Zvavitch

Problem 1. *Determine which of the following maps are linear. Prove your answers.*

(a) $F\colon \mathbb{R}^3 \to \mathbb{R}^2$ *defined by* $F(x, y, z) = (x + y + 1, x + y + z)$.
(b) $G\colon \mathbb{R}^2 \to \mathbb{R}^3$ *defined by* $G(x, y) = (x - y, x + 2y, y - x)$.
(c) $H\colon \mathbb{R}^2 \to \mathbb{R}^2$ *defined by* $H(x, y) = (x^2 + y, x)$.

Problem 2. *Find* $\varphi(a, b)$ *where* $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ *is a linear map defined by* $\varphi(1, 1) = (1, 2)$ *and* $\varphi(0, 1) = (1, -1)$.

Problem 3. *For a linear map* $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ *defined by*

$$\varphi(x, y, z) = (2x + y - z, x + y + z, 3x + y - 3z)$$

find the basis for the kernel and the image of φ.

Problem 4. *Find* $\varphi(a, b)$ *where* $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ *is a linear map defined by* $\varphi(1, 2) = (2, -1)$ *and* $\varphi(3, 1) = (1, 1)$.

Problem 5. *Show that the map* $\varphi\colon M_n(\mathbb{R}) \to M_n(\mathbb{R})$ *defined by* $\varphi(A) = A^T$ *is a linear transformation.*

Problem 6. *Show that if the linear transformation* $\varphi\colon V \to W$ *is onto, then*

$$\dim V \geq \dim W.$$

Problem 7. *Show that if the linear transformation* $\varphi\colon V \to W$ *is one-to-one, then*

$$\dim V \leq \dim W.$$

Problem 8. *Find a linear map* $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ *whose image is spanned by* $(1, 1, -2)$ *and* $(2, 3, -1)$.

Problem 9. *Find a linear map* $\varphi\colon \mathbb{R}^4 \to \mathbb{R}^3$ *whose kernel is spanned by* $(1, -1, 3, -2)$ *and* $(2, -3, 4, -1)$.