

**Theory of Numbers**  
**Home Work 11, due Thursday, April 30.**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** Find all solutions of  $x^2 \equiv 1 \pmod{15}$ . What conclusion you may make out of this example?

**Problem 2.** Solve the following quadratic congruences:

- $x^2 + 7x + 10 \equiv 0 \pmod{10}$ .
- $5x^2 + 6x + 1 \equiv 0 \pmod{23}$ .

**Problem 3.** Prove that equation  $6x^2 + 5x + 1 \equiv 0 \pmod{p}$  has a solution for all prime  $p$ , even though the equation  $6x^2 + 5x + 1 = 0$  has no solution in integers.

**Problem 4.** Show that 3 is quadratic residue of 23, but a non-residue of 31.

**Problem 5.** Let  $a$  be a quadratic residue of the odd prime  $p$ . Prove that  $a$  is not a primitive root of  $p$ .

**Problem 6.** Let  $a$  be a quadratic residue of the odd prime  $p$ . Show that  $p - a$  is a quadratic residue or non-residue of  $p$  depending on whether  $p$  is congruent to either 1 or 3 modulo 4.