Theory of Numbers
Home Work 2, due Thursday, January 29.
Instructor: Prof. Artem Zvavitch

Problem 1. *Please decide which of the following sternest are true and which are false. For true ones provide a proof for false ones give a counterexample,* $a, b, c \in Z$

- *If* $a|b$ *then* $a|bc$.
- *If* $a|b$ *and* $a|c$ *then* $a^2|bc$.
- *If* $a|b$ *and* $b|c$, *then* $a|\frac{c}{b}$.
- *If* $a|d$ *and* $b|c$ *then* $(a+b)|(d+c)$.
- *If* $a|d$ *and* $b|c$ *then* $ab|dc$.
- *If* $a|d$ *and* $a|c$ *then* $a|(d+c)$.
- *If* $a|(d+c)$ *the* $a|d$ *or* $a|c$.
- *If* $a, b > 0$ *and* $a|b^2$, *then there is* $x \in \mathbb{Z}$ *such that* $a = x^2$.

Problem 2. *Prove that for all* $a \in \mathbb{Z}$: $6|a(a^2+11)$.

Problem 3. *Prove that of* $gcd(a,b) = 1$ *and* $c|(a+b)$, *then* $gcd(a,c) = gcd(b,c) = 1$.

Problem 4. *Prove that if* $gcd(a,b) = 1$ *then* $gcd(a^2, b^2) = 1$.

Problem 5. *Find* x *and* y *such that*

$$gcd(1769, 2378) = 1769x + 2378y.$$

Problem 6. *Assume* d *is common devisor of* a *and* b, *please, prove that then* $d = gcd(a,b)$ *iff* $gcd(a/d, b/d) = 1$.

Problem 7. *Find* $gcd(a^2+b^2, a+b)$, *where* $a, b \in \mathbb{N}$ *and* $gcd(a,b) = 1$.