Theory of Numbers
Home Work 4, due Thursday, February 12.
Instructor: Prof. Artem Zvavitch

Problem 1. *Assume* p *is prime and* $p|a$ *prove that* $p^n|a^n$, *where* $a, n \in \mathbb{N}$.

Problem 2. *Assume* p *is prime and* $p|a^n$, *prove that* $p|a$, *where* $a, n \in \mathbb{N}$. *Would this statement be still true if we allow* p *to be a composite number?*

Problem 3. *Prove that* $\sqrt{3}$ *is an irrational number.*

Problem 4. *Assume a prime number* p *can be written as* $p = 5n+1$, *where* $n \in \mathbb{N}$, *show that* $p - 1 = 10m$, *where* $m \in \mathbb{N}$.

Problem 5. *Assume* $gcd(a, b) = p$ *and* p *is a prime number. Find* $gcd(a^n, b^m)$, $n, m \in \mathbb{N}$, $n, m > 1$.

Problem 6. *Find prime factorization of* 10152.

Problem 7. *Prove that if* $m, n \in \mathbb{N}$ *and* $\sqrt[n]{m}$ *is rational, then* $\sqrt[n]{m}$ *must be an integer.*

Problem 8. *Now, show that* $\sqrt[n]{n}$ *is irrational.*

Problem 9. *Assume* p_n *is nth prime number prove:*

- $p_n > 2n - 1$ *for* $n \geq 5$.
- $p_1 \cdot p_2 \cdot \cdots \cdot p_n + 1$ *is never a perfect square.*
- *The sum*
$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n}$$
is never an integer.