Theory of Numbers
Home Work 5, due Thursday, February 24.
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that if* $a \equiv b$ *(mod* n*) then* $gcd(a,n) = gcd(b,n)$.

Problem 2. *Is it true that if* $a^2 \equiv b^2$ *(mod* n*) and* $a, b > 0$ *then* $a \equiv b$ *(mod* n*).*

Problem 3. *Find the reminder of* 5^{1000} *decided by* 6.

Problem 4. *Use induction to prove that for any odd number* a *and* $n \geq 1$

$$a^{2^n} = 1(mod\ 2^{n+2}).$$

Problem 5. *Find the last two digits of the number* 9^{9^9}.

Problem 6. *Can a number* N*, sum of whose digits is* 21345*, be a square of a natural number?*

Problem 7. *Show that* 2^n *divides an integer* N *if and only if* 2^n *divides the number made up of the last* n *digits of* N*. (I would first play with* $2 = 2^1$ *and* $4 = 2^2$ *and after would do the general case)*

Problem 8. *Show that for any prime* $p > 3$

$$13 | 10^{2p} - 10^p + 1.$$