

Theory of Numbers
Home Work 6, due Thursday, March 5.
Instructor: Prof. Artem Zvavitch

Problem 1. *A $3n$ -digit number has all of its digits equal to 1. Use induction to show that this integer is divisible by 37.*

Problem 2. *Can a number, whose digits are one 1, two 2's, three 3's, . . . , nine 9's, written in some order, be a square of a natural number?*

Problem 3. *It is true that if*

$$a \equiv b \pmod{n}$$

then

$$a^2 \equiv b^2 \pmod{n^2}?$$

Problem 4. *Prove that $25^{99} + 14^{100}$ is divisible by 13.*

Problem 5. *Solve the following congruences:*

- $5x = 2 \pmod{26}$
- $34x = 60 \pmod{98}$
- $42x = 15 \pmod{14}$

Problem 6. *Find an integer having the remainders 2, 3, 4, 5 when divided by 3, 4, 5, 6, respectively.*

Problem 7. *Find a solution of the system of congruences:*

$$\begin{cases} 3x + 4y = 5 \pmod{13} \\ 2x + 5y = 7 \pmod{13} \end{cases}$$