Theory of Numbers
Home Work 7, due Thursday, March 12.
Instructor: Prof. Artem Zvavitch

Problem 1. *Assume that $p > 2$ is a prime, prove that*

$$1^p + 2^p + 3^p + ... + (p-2)^p + (p-1)^p \equiv 0 (mod\ p)$$

Problem 2. *Assume that $p > 2$ is a prime and $k \in [1, p-1]$ is an integerr, then*

$$\binom{p-1}{k} \equiv (-1)^k (mod\ p)$$

Problem 3. *Assume $p \neq q$ are primes prove that*

$$p^{q-1} + q^{p-1} \equiv 1 (mod\ pq)$$

Problem 4. *Consider a prime number p such that $M_p = 2^p - 1$ is composite. Prove that then M_p is pseudoprime.*

Please, start this AFTER we discuss Wilson's theorem

Problem 5. *If p is a prime, prove that for any $a \in \mathbb{Z}$*

$$p | a^p + (p-1)! a \text{ and } p | (p-1)! a^p + a.$$

Problem 6. *Assume that p and $p+2$ is the a pair of twin primes, then*

$$4((p-1)! + 1) + p \equiv 0 (mod\ p(p+2)).$$