Theory of Numbers
Home Work 8, due Thursday, March 19.
Instructor: Prof. Artem Zvavitch

Problem 1. *Compute* $\tau(1258)$ *and* $\sigma(1258)$.

Problem 2. *Does there exists* n *such that* $\sigma(n) = 9$*?*

Problem 3. *Check if the following statements are true or false (do not forget to provide a proof for true statements):*

- $\sigma(m + n) = \sigma(m) + \sigma(n)$.
- $\sigma(4k + 2) = 3\sigma(2k + 1)$.
- *If* $m \neq n$ *then* $\sigma(n) \neq \sigma(m)$.

Problem 4. *Prove that*

$$\sum_{d|n} \frac{1}{d} = \frac{\sigma(n)}{n},$$

for all positive n. *Also show that for all positive* n

$$\frac{\sigma(n!)}{n!} \geq \sum_{i=1}^{n} \frac{1}{i}.$$

Problem 5. *Let* $\omega(n)$ *denote the number of distinct prime divisors of* $n > 1$, *for example,* $\omega(60) = \omega(2^2 * 3 * 5) = 3$*:*

- *Show that* $2^{\omega(n)}$ *is a multiplicative function.*
- *For a positive integer* n *prove*

$$\tau(n^2) = \sum_{d|n} 2^{\omega(d)}.$$

Problem 6. *Prove that for each integer* n

$$\mu(n)\mu(n + 1)\mu(n + 2)\mu(n + 3) = 0$$

also show that for any integer $n \geq 3$

$$\sum_{k=1}^{n} \mu(k!) = 1.$$