

Theory of Numbers
Home Work 9, due Thursday, April 9.
Instructor: Prof. Artem Zvavitch

Problem 1. *The Mangoldt function Λ is defined by*

$$\Lambda(n) = \begin{cases} \log p, & \text{if } n = p^k, \text{ where } p \text{ is a prime and } k \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Prove that

$$\sum_{d|n} \Lambda(d) = \log n.$$

Next show that

$$\Lambda(n) = \sum_{d|n} \mu(n/d) \log d.$$

Problem 2. *Prove that*

- $\phi(2n) = \phi(n)$ for odd n .
- $\phi(2n) = 2\phi(n)$ for even n .
- $\phi(3n) = 3\phi(n)$ iff $3|n$.
- $\phi(3n) = 2\phi(n)$ iff $3 \nmid n$.
- That there are infinitely many integers n for which $\phi(n)$ is a perfect square (I would play with powers of 2).
- $\phi(n) \in [\frac{1}{2}\sqrt{n}, n]$, for any positive n .

Problem 3. *Use Euler theorem to prove that for all a*

$$a^{33} \equiv a \pmod{4080}.$$

Problem 4. *If m and n are relatively positive prime integers, prove that*

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}.$$

Problem 5. *Use Euler's theorem to find the last digit of 3^{100} .*