

Asymptotic Theory of Finite Dimensional Normed Spaces.
Home Works 4, due Wednesday, October 13
Instructor: Prof. Artem Zvavitch

Problem 1. *Please, show that concentration of Lipschitz function around its expectation imply concentration around the median. Please also show that concentration of Lipschitz functions imply almost isoperimetric inequality on S^{n-1} (do not worry about the constants!).*

Problem 2. *Consider a continuous function f on S^{n-1} . Let*

$$\mathcal{R}f(\xi) = \int_{S^{n-1} \cap \xi} f(\theta) d\theta \text{ and } \text{Cos}f(\xi) = \int_{S^{n-1}} |\theta \cdot \xi| f(\theta) d\theta$$

be respectively Radon and Cosine transforms of f . Prove that for any continuous function g

$$\int_{S^{n-1}} \mathcal{R}f \times g d\theta = \alpha \int_{S^{n-1}} f \times \mathcal{R}g d\theta \text{ and } \int_{S^{n-1}} \text{Cos}f \times g d\theta = \beta \int_{S^{n-1}} f \times \text{Cos}g d\theta,$$

where α and β are absolute constants, i.e. independent of f and g , also, please find those constants.

Problem 3. *We proved in class, that a cardinality of best ε -net on S^{n-1} with respect to Euclidean distance can not be too big. Please, show that cardinality of every ε -net (with respect to Euclidean distance) is bigger than $(\frac{1}{\varepsilon})^{n-1}$.*

Problem 4. *Let g_1, \dots, g_n be independent identically distributed $N(0, 1)$ -random variables. Consider $a \in S^{n-1}$. Find distribution of $\sum_{i=1}^n a_i g_i$.*

Problem 5. *Using previous problem! Find $(\mathbb{E}|\sum_{i=1}^n a_i g_i|^p)^{1/p}$.*