

Introduction to Partial Differential Equations.
Home Work 2, due Wednesday, February 11.
Instructor: Prof. Artem Zvavitch

Problem 1. Solve heat equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, $x \in [0, L]$, $t > 0$. subject to $u(0, t) = 0$, $u(L, t) = 0$ and

$$u(x, 0) = 2 \cos \frac{3\pi x}{L}.$$

Problem 2. Solve heat equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to $\frac{\partial u}{\partial x} u(0, t) = 0$, $\frac{\partial u}{\partial x} u(L, t) = 0$ and initial condition

$$u(x, 0) = \begin{cases} 1 & \text{for } 0 < x < L/2 \\ 2 & \text{for } L/2 < x < L \end{cases}$$

Problem 3. Solve the eigenvalue problem

$$\phi''(x) = -\lambda \phi(x)$$

subject to $\phi(0) = \phi(2\pi)$ and $\phi'(0) = \phi'(2\pi)$.

Problem 4. Solve Laplace's equation $\Delta u(x, y) = 0$ inside rectangle $[0, L] \times [0, H]$, subject to

$$\frac{\partial u}{\partial x}(0, y) = 0, u(L, y) = g(y), u(x, 0) = 0, u(x, H) = 0.$$

Problem 5. Solve Laplace's equation outside a circular disk of radius a (i.e. $r \geq a$) subject to the boundary condition $u(a, \theta) = \ln 2 + 4 \cos 3\theta$.