Introduction to Partial Differential Equations.
Home Work 3, due Thursday, February 18.
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that the Fourier series operation is linear: that is, show that the Fourier series of $c_1 f + c_2 g$ is the sum of c_1 times Fourier series of f and c_2 times Fourier series of g.*

Problem 2. *Consider function*

$$f(x) = \begin{cases} x \text{ for } x < 0 \\ 1 \text{ for } x > 0 \end{cases}$$

- *Explain why the Fourier series of $f(x)$ (on the intreval $[-L, L]$) converges.*
- *Sketch the Fourier series of $f(x)$.*
- *Compute the Fourier coefficients.*
- *Compute even and odd parts of $f(x)$.*
- *Sketch the sine series of $f(x)$.*
- *Sketch the cosine series of $f(x)$.*

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Problem 3. *Please, solve the following* **non-homogeneous** *problem*

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + e^{-t} + e^{-2t}\cos\frac{3\pi x}{L}, \text{ where } 2 \neq k(3\pi/L)^2,$$

subject to

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0 \text{ and } u(x,0) = f(x).$$

Idea: use for the solution as a Fourier cosine series. Justify all differentiation of infinite series (assume appropriate conditions).