21001, Section 01, Linear Algebra and applications
Final EXAM FOR FUN! TRY IT FOR FREE !!!
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GOOD LUCK!!!

Problem 1. *Let:*

$$\begin{pmatrix} -2 & 0 & -1 & 2 \\ 6 & 2 & -4 & 4 \\ 0 & 0 & -8 & 1 \\ -3 & 0 & 0 & 0 \end{pmatrix}.$$

Find: $det(A)$, $det(A^2)$, $det(A^{-1})$, $det((A^T)^2)$.

Problem 2. *Find the inverse of the given matrix,*

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Problem 3. W *is the set of all* 3×3 *matrices of the form*

$$\begin{pmatrix} a & 2a & 0 \\ -a & a & -3a \\ a^2 & 0 & 0 \end{pmatrix}.$$

Is it true that W *is a subspace of* $M_{3,3}$

Problem 4. *Determine whether the set* $W = \{(x, -y, x+y) : x, y \text{ are real numbers}\}$ *is a subspace of* $\mathbb{R}^3$ *with the standard operations.*

Problem 5. *Check if the set* $S = \{(0, 1, 2), (1, 1, 0), (1, -3, 2)\}$ *spans* $\mathbb{R}^3$.

Problem 6. *If it is possible, write vector* $\mathbf{w} = (1, 3, 1)$ *as a linear combination of vectors* $(1, 1, 2), (1, 1, 0), (1, 0, 1)$.

Problem 7. *Determined if the set of vectors*

$$S = \{1 + x + x^2, x^2 - 2x, 1 + 2x - 2x^2\}$$

is linearly dependent.

Problem 8. *Show that* $\{-2, x - 1, -x^2 + x\}$ *is a basis of* P_2.

Problem 9. *Consider matrix* A *such that*

$$A = \begin{bmatrix} -11 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & -1 \\ -2 & 1 & 2 & -1 & 0 \end{bmatrix}.$$

Find

- *Basis and dimension of the row space of* A.

- *Basis and dimension of the column space of A.*
- *$rank(A)$.*
- *Null set of A.*
- *$nullity(A)$.*

Problem 10. *Determine whether the nonhomogeneous system of equations is consistent. If yes, then fined all solutions (i.e. write the solution in the form $x = x_h + x_p$, where x_h is a solution of corresponding homogenous system and x_p is a particular solution).*

$$x_1 + x_2 + x_3 + x_4 = 1$$
$$-x_1 + 3x_2 - x_3 + 2x_4 = -1$$
$$x_1 - 2x_2 + x_3 + 3x_4 = 1$$

Problem 11. *Find the transition matrix from basis B to basis B' if*

$$B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)$$

and

$$B = \{(1, 2, -1), (-1, 0, 1), (0, 1, 1).$$

Also find $[x]_{B'}$ if $[x]_B = (-1, 2, -3)$.

Problem 12. *Please, find eigenvectors and eigenvalues of the following matrix.*

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 3 & 3 \end{bmatrix}$$