

**21001, Section 01, Linear Algebra and applications**  
**HW 5, DUE Friday, October 22**  
**Instructor: Prof. Artem Zvavitch**  
**GOOD LUCK!!!**

**Problem 1.** Consider matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -2 & 5 \\ 0 & 1 & 1 \end{bmatrix}.$$

Find  $\det(A)$ ;  $\det(A^T)$ ;  $\det(A^T A)$ ;  $\det(A^{10})$ ;  $\det(A^{-3})$  (HINT: THINK! and you will not need to do any calculations!).

**Problem 2.** Find the characteristic equation and the eigenvalues of the matrix

$$\begin{bmatrix} 4 & 0 & -2 \\ 0 & -2 & 5 \\ 0 & 4 & 6 \end{bmatrix},$$

**Problem 3.** Let  $\mathbf{v} = (1, -2, 3, 0, 2)$  and  $\mathbf{u} = (2, -1, -3, 1, -2)$ . Please, find  $3\mathbf{v} - 2\mathbf{u}$

**Problem 4.** Determine whether the set, together with indicated operations, is a vector space.

- (1) The set  $\{(x, -x) : x \text{ is a real number}\}$  with the standard operations.
- (2) The set  $\{(x, y) : x, y \text{ are a real numbers such that } x > y\}$  with the standard operations.