

**21001, Section 01, Linear Algebra and applications**  
**HW 5, DUE Wednesday, March 9**  
**Instructor: Prof. Artem Zvavitch**  
**GOOD LUCK!!!**

**Problem 1.** Let  $\mathbf{v} = (1, -2, 3, 0, 2)$  and  $\mathbf{u} = (2, -1, -3, 1, -2)$ . Please, find  $3\mathbf{v} - 2\mathbf{u}$

**Problem 2.** Determine whether the set, together with indicated operations, is a vector space.

- (1) The set  $\{(x, -x) : x \text{ is a real number}\}$  with the standard operations.
- (2) The set  $\{(x, y) : x, y \text{ are a real numbers such that } x > y\}$  with the standard operations.

**Problem 3.** Consider the set  $W$  consisting of all  $2 \times 3$  matrices of the form

$$A = \begin{bmatrix} a & -b & 0 \\ 0 & c & b \end{bmatrix},$$

where  $a, b, c$  are arbitrary real numbers. Show that  $W$  is a subspace of  $M_{2,3}$  (all  $2 \times 3$  matrices).

**Problem 4.** Is  $W = \{(x, x+1, x+2) : x \text{ is a real number}\}$  a subspace of  $\mathbb{R}^3$  ?

**Problem 5.** Let  $A$  be a  $2 \times 2$  matrix prove that  $W = \{x \in \mathbb{R}^2 : A\mathbf{x} = \mathbf{0}\}$  is a subspace of  $\mathbb{R}^2$ .

**Problem 6.** Let  $A$  be a  $2 \times 2$  matrix show that if  $\mathbf{b} \neq \mathbf{0}$  ( $\mathbf{b}$  is not zero vector) then  $W = \{x \in \mathbb{R}^2 : A\mathbf{x} = \mathbf{b}\}$  is NOT a subspace of  $\mathbb{R}^2$ .