21001, Section 01, Linear Algebra and applications
HW 9, DUE Wednesday, April 20
Instructor: Prof. Artem Zvavitch
GOOD LUCK!!!

Problem 1. *Fin a basis for the subspace of* $\mathbb{R}^4$ *spanned by vectors*

$$(2,4,-3,-6),(7,14,-6,-3),(-2,-4,1,-2),(1,2,-1,-1).$$

Problem 2. *Determine whether the nonhomogeneous system of equations is consistent. If yes, then fined all solutions (i.e. write the solution in the form* $x = x_h + x_p$*, where* x_h *is a solution of corresponding homogenous system and* x_p *is a particular solution).*
a)

$$\begin{aligned} x_1 + 4x_2 + 3x_3 + 2x_4 &= 10 \\ -5x_1 + 6x_2 + 11x_3 + 12x_4 &= 24 \\ 3x_1 - 2x_2 + x_3 + 5x_4 &= 7 \end{aligned}$$

b)

$$\begin{aligned} -x + 2y + 2z &= -1 \\ 3x - 6y + 4z &= 3 \\ 5x - 10y - 3z &= 5 \end{aligned}$$

Problem 3. *Determine whether* b *is in the column space of* A*. If it is, write* b *as a linear combination of the column vectors of* A*.*
a)

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -3 \\ -2 & 4 & 2 & -10 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ -1 \\ -8 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

Problem 4. *Let* A *be an* $n \times n$ *matrix such that* $\det(A) \neq 0$*. Find* $rank(A^{-1})$*.*