

MATH-57091 Probability and Statistics for High-School  
Teachers.

Home Work 1, due on Monday SEPTEMBER 10,  
Instructor: Prof. Artem Zvavitch

**Problem 1.** *Prove that*

$$S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$$

*also prove de Morgan's law:*

$$\left( \bigcap_n S_n \right)^c = \bigcup S_n^c.$$

**Problem 2.** *If two fair dice are tossed,*

- *what is the probability that one of the dice shows 5?*
- *what is the probability that the sum is 6?*
- *what is the probability that one of the dice shows 1 given that the sum of all three is 6?*

**Problem 3.** *Use axioms to prove that*

- $P(A \cup B) \geq \max\{P(A), P(B)\},$
- $P(A \cap B) \leq \min\{P(A), P(B)\}.$

**Problem 4.** *Suppose cards numbered one through ten are placed in a hat, mixed up, and then one of the cards is drawn. If we are told that the number on the drawn card is at least five, then what is the conditional probability that it is ten?*

**Problem 5.** *Suppose an urn contains seven black balls and five white balls. We draw two balls from the urn without replacement (i.e. we first take the first ball and after the second one WITHOUT putting the first ball back!). Assuming that each ball in the urn is equally likely to be drawn, what is the probability that both drawn balls are black?*

**Problem 6.** *Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly select a hat. What is the probability that none of the three men selects his own hat?*