MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 1, due on Wednesday SEPTEMBER 3.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Prove that*

- $(S^c)^c = S$.
- $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$.
- *also prove de Morgan's law:*

$$\left(\bigcap_n S_n\right)^c = \bigcup S_n^c.$$

Problem 2. *If two fair dice are tossed,*

- *what is the probability that one of the dice shows 5?*
- *what is the probability that the sum is 6?*

Problem 3. *Use axioms to prove that*

- $P(A \cup B) \geq \max\{P(A), P(B)\}$,
- $P(A \cap B) \leq \min\{P(A), P(B)\}$.

Problem 4. *Assume* $P(A \cup B) = P(A) + P(B)$*, find* $P(A \cap B)$.

Problem 5. *Suppose cards numbered one through ten are placed in a hat, mixed up, and then one of the cards is drawn. Please, describe the sample space. Please, find the probability that the number is greater then* 6*? What is the probability that the number is less or equal to* 3.5*? What is the probability that the number is ether less or equal to* 3.5 *or greater then* 6 *or both?*

Problem 6. *Suppose an urn contains seven black balls and five white balls. We draw two balls from the urn without replacement (i.e. we first take the first ball and after the second one WITHOUT putting the first ball back!). Assuming that each ball in the urn is equally likely to be drawn, what is the probability that both drawn balls are black?*

Problem 7. *Romeo and Juliet has a meeting at given time but each arrive a "bit" late. Say each of them may be up to one hour late. We assume that all pairs of delays are equally likely. The first who arrive will wait for* 15 *minutes and then leave, if the other one is not at the place. What is the probability that they will meet but the first who arrive will have to wait for at least* 5 *minutes?*

Problem 8. *Assume* $P(A \cap B) = 0$ *is it true that then* $A \cap B = \emptyset$*?*