MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 2, due on Monday SEPTEMBER 17,
Instructor: Prof. Artem Zvavitch

Problem 1. *Suppose that* $\mathbb{P}(E) = 0.6$. *What can you say about* $\mathbb{P}(E|F)$ *when*

- E *and* F *are mutually exclusive? (i.e.* $E \cap F = \emptyset$*)?*
- $E \subset F$*?*
- $F \subset E$*?*

Problem 2. *A laboratory blood test is* 95 *percent effective in detecting a certain decease when it is, in fact, present. However, the test also yields a "false positive" result in* 1 *percent of the healthy persons tested (i.e. if the healthy person tested then with probability* 0.01*, the test result will imply he has the disease). If* 0.5 *percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?*

Problem 3. *Stores* A*,* B *and* C *have* 50*,* 75 *and* 100 *employees and respectively* 50*,* 60 *and* 70 *percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman What is the probability that she works in store* C*?*

Problem 4. *There four red cubes, six red balls and six yellow cubes in a box. How many yellow balls should you put in the box to make color and shape independent when the object from the box is selected at random?*

Special note for Problem 5: The definition of independent can be extended to more then two event. The events $A_1, A_2, \dots, A_n$ are independent if for EVERY collection $A_{1'}, A_{2'}, \dots, A_{i'}$ (for $2 \le i \le n$) of those events we have

$$\mathbb{P}(A_{1'} \cap A_{2'} \cap \dots \cap A_{i'}) = \mathbb{P}(A_{1'})\mathbb{P}(A_{2'}) \dots \mathbb{P}(A_{i'}).$$

So "ideology" is that $A_1, \dots, A_n$ are independent if knowledge of the occurrence of ANY of those events has no effect on the probability of any other events.

Problem 5. *(Pairwise Independent events that are not Independent) Le a ball be drawn from an urn containing four balls, numbered* $1, 2, 3, 4$*. All four outcomes are equally likely. Let* $E = \{1, 2\}$ *(i.e the the drawn ball have number* 1 *or* 2 *on it),* $F = \{1, 3\}$ *and* $G = \{1, 4\}$*.*

Please show that any pair of those events is independent, i.e.

$$\mathbb{P}(E \cap F) = \mathbb{P}(E)\mathbb{P}(F)$$

$$\mathbb{P}(E \cap G) = \mathbb{P}(E)\mathbb{P}(G)$$

$$\mathbb{P}(G \cap F) = \mathbb{P}(G)\mathbb{P}(F).$$

But, that those events are not independent:

$$\mathbb{P}(E \cap F \cap G) \neq \mathbb{P}(E)\mathbb{P}(F)\mathbb{P}(G).$$