

MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 2, due on Wednesday SEPTEMBER 12,
Each problem 11 points (yes you have 5 points extra)
Instructor: Prof. Artem Zvavitch

Problem 1. Suppose that $\mathbb{P}(E) = 0.6$. What can you say about $\mathbb{P}(E|F)$ when

- E and F are mutually exclusive? (i.e. $E \cap F = \emptyset$)?
- $E \subset F$?
- $F \subset E$?

Problem 2. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result in 1 percent of the healthy persons tested (i.e. if the healthy person tested then with probability 0.01, the test result will imply he has the disease). If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

Problem 3. Stores A , B and C have 50, 75 and 100 employees and respectively 50, 60 and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C ?

Problem 4. There four red cubes, six red balls and six yellow cubes in a box. How many yellow balls should you put in the box to make color and shape independent when the object from the box is selected at random?

Special note for Problem 5: The definition of independent can be extended to more than two event. The events A_1, A_2, \dots, A_n are independent if for EVERY collection $A_{1'}, A_{2'}, \dots, A_{i'}$ (for $2 \leq i' \leq n$) of those events we have

$$\mathbb{P}(A_{1'} \cap A_{2'} \cap \dots \cap A_{i'}) = \mathbb{P}(A_{1'})\mathbb{P}(A_{2'}) \dots \mathbb{P}(A_{i'}).$$

So "ideology" is that A_1, \dots, A_n are independent if knowledge of the occurrence of ANY of those events has no effect on the probability of any other events.

Problem 5. (Pairwise Independent events that are not Independent) Let a ball be drawn from an urn containing four balls, numbered 1, 2, 3, 4. All four outcomes are equally likely. Let $E = \{1, 2\}$ (i.e. the drawn ball have number 1 or 2 on it), $F = \{1, 3\}$ and $G = \{1, 4\}$.

Please show that any pair of those events is independent, i.e.

$$\mathbb{P}(E \cap F) = \mathbb{P}(E)\mathbb{P}(F)$$

$$\mathbb{P}(E \cap G) = \mathbb{P}(E)\mathbb{P}(G)$$

$$\mathbb{P}(G \cap F) = \mathbb{P}(G)\mathbb{P}(F).$$

But, that those events are not independent:

$$\mathbb{P}(E \cap F \cap G) \neq \mathbb{P}(E)\mathbb{P}(F)\mathbb{P}(G).$$