

MATH-57091 Probability and Statistics for High-School  
Teachers.

Home Work 3, due on Wednesday SEPTEMBER 19,  
Instructor: Prof. Artem Zvavitch

**Problem 1. (25 points)** Suppose that  $X$  is a random variable that takes on one of the values  $-1, 0, 1$  and  $2$ . If

$$\mathbb{P}(X = -1) = .2, \mathbb{P}(X = 0) = .3 \text{ and } \mathbb{P}(X = 1) = .4,$$

Find

- $\mathbb{P}(X = 2)$ .
- $\mathbb{P}(X \leq 1)$ .
- $\mathbb{P}(X > 0)$ .
- $\mathbb{P}(-\frac{1}{2} \leq X < 2)$ .
- $\mathbb{E}X$ .
- *Distribution (Probability Mass function) of  $X^2$ . (Probability mass function is a function that gives the probability that a discrete random variable is exactly equal to some value. For example:  $\mathbb{P}(X = -1) = .2, \mathbb{P}(X = 0) = .3$  and  $\mathbb{P}(X = 1) = .4$ . If now we consider a new random variable  $Y = X + 3$  we would get  $\mathbb{P}(Y = 2) = .2, \mathbb{P}(Y = 3) = .3$  and  $\mathbb{P}(Y = 4) = .4$ . The question is to find the Probability mass function for  $X^2$ ).*
- $\mathbb{E}X^2$ .
- $\mathbb{E}2^X$ .

Also let  $Y$  be a random variable that takes on one of the values  $-2, 0, 3$ . If

$$\mathbb{P}(Y = -2) = .2, \mathbb{P}(Y = 0) = .3 \text{ and } \mathbb{P}(Y = 3) = .5,$$

find

- $\mathbb{E}Y$ ,
- $\mathbb{E}(X + Y)$ . (Hint: try to do it directly, but after see Lecture 3.2)

**Problem 2. (10 points)** An insurance agent has two clients each of whom has a life insurance policy that pays 100000 dollars upon death. Their probabilities of dying this year are .05 and .1. Let  $X$  denote the total amount of money that will be paid this year to the clients beneficiaries. Assuming that the event that client 1 dies is independent of the event that client 2 dies, determine the probability distribution (probability mass function) of  $X$  also  $\mathbb{E}X$ .

**Problem 3. (10 points)** Let  $\mathbb{E}X = \mathbb{E}Y = 0$  is it true that then  $\mathbb{E}XY = 0$ .

**Problem 4. (10 points)** Let  $Y$  be a Geometric Random Variable with parameter  $p$ , find  $\mathbb{E}Y$ .