MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 4, due on Wednesday, September 28,
Instructor: Prof. Artem Zvavitch

Problem 1. (8 points) *A building constructor has sent in bids for three jobs. If the constructor obtains these jobs, they will yield respective profits* $20, 25$ *and* 40 *(in \$* 1000*). On the other hand, for each job the constructor does not win, he will incur a loss (due to time and money already spent in making the bid) of* 2*. If the probabilities that the constructor will get these jobs are, respectively* $.3, .6$ *and* $.2$*, what is the expected total profit?*

Problem 2. (8 points) *A fair bet is one in which the expected gain is equal to* 0*. If you bet* 1 *unit on a number in roulette, then you will gain* 35 *units if the number appears and will lose* 1 *unit if it does not. If the roulette wheel is perfectly balanced, then the probability that your number will appear is* $1/38$*. What is the expected gain for* 1 *unit? Is it a fair bet?*

Problem 3. (8 points) *Suppose that you collect coupons! Suppose that each coupon obtained is, independent of what has been previously obtained, equally likely to be any of* m *types. Find the expected number of coupons one need to obtain in order to have at least one of each type.* **HINT:** *Let* X *be the number needed. It may help to present* X *by*

$$X = \sum_{i=1}^{m} X_i$$

where each X_i *is geometric random variable (and represent how many coupons you need, AFTER you collected* $i-1$ *different coupons and trying to collect a coupon of a new type, i.e. to have* i *different coupons. For example* $X_1 = 1$*,* X_2 *is just a bit more tricky...).*

Problem 4. (12 points) *Artem plays with his daughter Maya a game: Maya rolls a die, if number* 3 *appears she gets a dollar, any other number appears she gets nothing. Assume they plan to play* 20 *times. Let* X *be a random variable representing the amount of money Maya may win.*

- *What distribution should you use to model* X*? Write the formula for distribution (probability mass function) of* X*.*
- *Find* $\mathbb{E}X$*,* $\mathrm{Var}(X)$ *and* $\mathrm{SD}(X) = \sqrt{Var(X)}$*.*
- *Use calculator to compute (approximately!)* $\mathbb{P}(X = k)$*, for* $k = 0, 1, \dots, 20$ *and make a graph chart.*

Assume, Artem decided to make the game more fun and each time Maya play she needs to pay 25 *cents. Again they plan to play* 20 *times and let* Y *is an amount Maya may get as an income after those* 20 *games. Please, find* $\mathbb{E}Y$*,* $\mathrm{Var}(Y)$*, and* $\mathrm{SD}(Y)$*.* **Never play such games with your kids :)**

Problem 5. (12 points) *Prove that* $\mathbb{E}X^2 \geq (\mathbb{E}X)^2$*, also explain when/if equality is possible.* **Hint: Play with** $\mathbb{E}(X - \mathbb{E}X)^2$**.**

Problem 6. (12 points) *Assume* X *and* Y *are independent discrete random variables. Assume* Y *takes only positive numbers is it true that (if yes, present a proof, if not show a counterexample):*

- $\mathbb{E}\frac{X}{Y} = \frac{\mathbb{E}X}{\mathbb{E}Y}$*?*
- $\mathbb{E}\frac{X}{Y} = (\mathbb{E}X)(\mathbb{E}\frac{1}{Y})$*?*