

MATH-57091 Probability and Statistics for High-School Teachers.

Home Work 4, due on Wednesday, September 26,

Instructor: Prof. Artem Zvavitch

**Problem 1. (8 points)** A building constructor has sent in bids for three jobs. If the constructor obtains these jobs, they will yield respective profits 20, 25 and 40 (in \$ 1000). On the other hand, for each job the constructor does not win, he will incur a loss (due to time and money already spent in making the bid) of 2. If the probabilities that the constructor will get these jobs are, respectively .3, .6 and .2, what is the expected total profit?

**Problem 2. (8 points)** A fair bet is one in which the expected gain is equal to 0. If you bet 1 unit on a number in roulette, then you will gain 35 units if the number appears and will lose 1 unit if it does not. If the roulette wheel is perfectly balanced, then the probability that your number will appear is  $1/38$ . What is the expected gain for 1 unit? Is it a fair bet?

**Problem 3. (8 points)** Suppose that you collect coupons! Suppose that each coupon obtained is, independent of what has been previously obtained, equally likely to be any of  $m$  types. Find the expected number of coupons one need to obtain in order to have at least one of each type. **HINT:** Let  $X$  be the number needed. It may help to present  $X$  by

$$X = \sum_{i=1}^m X_i$$

where each  $X_i$  is geometric random variable (and represent how many coupons you need, AFTER you collected  $i - 1$  different coupons and trying to collect a coupon of a new type, i.e. to have  $i$  different coupons. For example  $X_1 = 1$ ,  $X_2$  is just a bit more tricky...).

**Problem 4. (10 points)** Artem plays with his daughter Maya a game: Maya rolls a die, if number 3 appears she gets a dollar, any other number appears she gets nothing. Assume they plan to play 20 times. Let  $X$  be a random variable representing the amount of money Maya may win.

- What distribution should you use to model  $X$ ? Write the formula for distribution (probability mass function) of  $X$ .
- Find  $\mathbb{E}X$ ,  $\text{Var}(X)$  and  $\text{SD}(X) = \sqrt{\text{Var}(X)}$ .
- Use calculator to compute (approximately!)  $\mathbb{P}(X = k)$ , for  $k = 0, 1, \dots, 20$  and make a graph chart.

Assume, Artem decided to make the game more fun and each time Maya play she needs to pay 25 cents. Again they plan to play 20 times and let  $Y$  is an amount Maya may get as an income after those 20 games. Please, find  $\mathbb{E}Y$ ,  $\text{Var}(Y)$ , and  $\text{SD}(Y)$ . **Never play such games with your kids :)**

**Problem 5. (10 points)** Prove that  $\mathbb{E}X^2 \geq (\mathbb{E}X)^2$ , also explain when/if equality is possible. **Hint:** Play with  $\mathbb{E}(X - \mathbb{E}X)^2$ .

**Problem 6. (10 points)** Assume  $X$  and  $Y$  are independent discrete random variables. Assume  $Y$  takes only positive numbers is it true that (if yes, present a proof, if not show a counterexample):

- $\mathbb{E}\frac{X}{Y} = \frac{\mathbb{E}X}{\mathbb{E}Y}$ ?
- $\mathbb{E}\frac{X}{Y} = (\mathbb{E}X)(\mathbb{E}\frac{1}{Y})$ ?