

MATH-57091 Probability and Statistics for High-School
Teachers.

Home Work 5, due on Monday October 8,

Instructor: Prof. Artem Zvavitch

Problem 1. Let X be a uniform random variable over the interval $[3, 11]$. Please, find the density of X , cumulative distribution of X (i.e. $F_X(t) = \mathbb{P}(X < t)$), $\mathbb{E}X$ and $\text{Var}(X)$.

Definition: m is a median of random variable X if

$$\mathbb{P}(X \geq m) = \mathbb{P}(X \leq m) = \frac{1}{2}.$$

Problem 2. Let X be a uniform random variable over the interval $[3, 11]$. Please, find median of X .

Problem 3. Let the probability density of Y be given by

$$f_Y(x) = \begin{cases} c(4x - 2x^2) & \text{if } x \in (0, 2); \\ 0 & \text{otherwise.} \end{cases}$$

Please, find

- constant c .
- cumulative distribution of Y (i.e. $F_Y(t) = \mathbb{P}(Y < t)$).
- $\mathbb{P}(.5 < Y \leq 1.5)$.
- $\mathbb{E}Y$, $\text{Var}(Y)$ and $\mathbb{E}Y^3$.
- Find the median of Y , btw is it true that the median is always the same as expected value?

Problem 4. Let G be a $N(1, 4)$ (i.e. $\mu = 1$ and $\sigma^2 = 4$) normal random variable. Please use a table of $N(0, 1)$ (internet will help!) to approximate $\mathbb{P}(-1 < G \leq 3)$.