MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 5, due on Wednesday, October 5,
Instructor: Prof. Artem Zvavitch

Problem 1. (12 points) *Let X be a uniform random variable over the interval $[3, 11]$. Please, find the density of X, cumulative distribution of X (i.e. $F_X(t) = \mathbb{P}(X < t)$), $\mathbb{E}X$ and $\mathrm{Var}(X)$.*

Definition: m is a median of random variable X if

$$\mathbb{P}(X \geq m) = \mathbb{P}(X \leq m) = \frac{1}{2}.$$

Problem 2. (10 points) *Let X be a uniform random variable over the interval $[3, 11]$. Please, find the median of X.*

Problem 3. (15 points) *Let the probability density of Y be given by*

$$f_Y(x) = \begin{cases} c(4x - 2x^2) & \text{if } x \in (0, 2); \\ 0 & \text{otherwise.} \end{cases}$$

Please, find

- *constant c.*
- *cumulative distribution of Y (i.e. $F_Y(t) = \mathbb{P}(Y < t)$).*
- $\mathbb{P}(.5 < Y \leq 1.5)$.
- $\mathbb{E}Y$, $\mathrm{Var}(Y)$ *and* $\mathbb{E}Y^3$.
- *Find the median of Y, btw is it true that the median is always the same as expected value?*

Problem 4. (15 points) *Let $X_1, X_2, \dots, X_n$ be independent random variables, each having uniform distribution over $(0, 1)$. Let*

$$M = \max(X_1, X_2, \dots, X_2).$$

Show that the distribution function of M is

$$F_M(t) = P(M \leq t) = t^n, \text{ for } t \in (0, 1).$$

What is the density function of M?

Problem 5. (10 points) *Let G be a $N(1, 4)$ (i.e. $\mu = 1$ and $\sigma^2 = 4$) normal random variable. Please use a table of $N(0, 1)$ (internet will help!) to approximate $\mathbb{P}(-1 < G \leq 3)$.*