

MATH-57091 Probability and Statistics for High-School
Teachers.

Home Work 6, due on Monday October 8,

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Problem 1. Suppose we know that the the average salary of a person living in Moscow is \$900 for the month of January, 2012.

- If we select a random person in Moscow, approximate the probability that his salary in January was over \$1500?
- If we also know that the variance of the January, 2012 salaries is \$100. Approximate the probability that that a randomly selected person from Moscow had a salary between 700 and 1100 dollars in January 2012?

Problem 2. Prove Markov's inequality for a non-negative desecrate variable X .

Problem 3. Use Chebyshev's inequality to prove the **week law of large numbers**. namely, if X_1, X_2, \dots are independent random variables having the same mean and variance (i.e. $\mathbb{E}X_i = \mu$ and $\text{Var}(X_i) = \sigma^2$ for all $i = 1, 2, \dots$, then for all $\varepsilon > 0$:

$$\lim_{n \rightarrow \infty} P \left\{ \left| \frac{X_1 + X_2 + \dots + X_n}{n} - \mu \right| > \varepsilon \right\} = 0.$$

Problem 4. Suppose that X is a random variable with mean 10 and variance 15 what can we say about $\mathbb{P}(5 < X < 15)$?

PLEASE, STOP: CONTINUE ONLY AFTER LECTURE 6.2

Problem 5. (Normal Approximation to the Binomial): let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Next use the Central Limit Theorem to approximate $\mathbb{P}(X = 20)$. (Hint. Notice that $\mathbb{P}(X = 20) = \mathbb{P}(19.5 < X < 20.5)$).

Problem 6. The lifetime of a special type battery is a random variable with mean 40 hours and standard deviation 20 hours. Let X be a life time of a battery (A battery is used until failed!)

- Use Markovs inequality to estimate $\mathbb{P}(X \geq 80)$.
- Use Chebychevs inequality to estimate $\mathbb{P}(30 < X < 50)$.

Assuming a stockpile of 25 such batteries the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained. (**Hint:** Use Central Limit Theorem).