MATH-57091 Probability and Statistics for High-School Teachers.
Home Work 6, due on Wednesday October 12,
Instructor: Prof. Artem Zvavitch

Problem 1. *Suppose we know that the the average salary of a person living in Moscow is \$900 for the month of January, 2012.*

- **(5 points)** *If we select a random person in Moscow, approximate (find an upper bound) the probability that his/her salary in January was over \$1500?*
- **(5 points)** *If we also know that the variance of the January, 2012 salaries is \$100. Approximate (find an upper bound) the probability that that a randomly selected person from Moscow had a salary between 700 and 1100 dollars in January 2012?*

Problem 2. (10 points) *Prove Markov's inequality for a non-negative desecrate variable X.*

Problem 3. (10 points) *Use Chebyshev's inequality to prove the* **week law of large numbers**. *namely, if $X_1, X_2, \dots$ are independent random variables having the same mean and variance (i.e. $\mathbb{E}X_i = \mu$ and $Var(X_i) = \sigma^2$ for all $i = 1, 2, \dots,$ then for all $\varepsilon > 0$:*

$$\lim_{n\to\infty} P\left\{\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| > \varepsilon\right\} = 0.$$

Problem 4. (5 points) *Suppose that X is a random variable with mean* 10 *and variance* 15 *what can we say about $\mathbb{P}(5 < X < 15)$?*

PLEASE, STOP: CONTINUE ONLY AFTER LECTURE 6.2

Problem 5. (10 points) *(Normal Approximation to the Binomial): let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Next use the Central Limit Theorem to approximate $\mathbb{P}(X = 20)$. (Hint. Notice that $\mathbb{P}(X = 20) = \mathbb{P}(19.5 < X < 20.5)$).*

Problem 6. (10 points) *The lifetime of a special type battery is a random variable with mean 40 hours and standard deviation 20 hours. Let X be a life time of a battery (A battery is used until failed!)*

- *Use Markovs inequality to estimate $\mathbb{P}(X \geq 80)$.*
- *Use Chebychevs inequality to estimate $\mathbb{P}(30 < X < 50)$.*

Assuming a stockpile of 25 such batteries the lifetimes of which are independent, approximate the probability that over 1100 *hours of use can be obtained. (***Hint:** *Use Central Limit Theorem).*