

MATH-57091 Probability and Statistics for High-School
Teachers.

Home Work 6, due on Wednesday October 10,
Instructor: Prof. Artem Zvavitch

Problem 1. *Suppose we know that the the average salary of a person living in Moscow is \$900 for the month of January, 2012.*

- **(5 points)** *If we select a random person in Moscow, approximate (find an upper bound) the probability that his/her salary in January was over \$1500?*
- **(5 points)** *If we also know that the variance of the January, 2012 salaries is \$100. Approximate (find an upper bound) the probability that that a randomly selected person from Moscow had a salary, in January 2012, which does not belong to interval from 700 to 1100 dollars?*

Problem 2. (10 points) *Prove Markov's inequality for a non-negative discrete variable X .*

Problem 3. (10 points) *Use Chebyshev's inequality to prove the **weak law of large numbers**. namely, if X_1, X_2, \dots are independent random variables having the same mean and variance (i.e. $\mathbb{E}X_i = \mu$ and $\text{Var}(X_i) = \sigma^2$ for all $i = 1, 2, \dots$, then for all $\varepsilon > 0$:*

$$\lim_{n \rightarrow \infty} P \left\{ \left| \frac{X_1 + X_2 + \dots + X_n}{n} - \mu \right| > \varepsilon \right\} = 0.$$

Problem 4. (5 points) *Suppose that X is a random variable with mean 10 and variance 15 what can we say about $\mathbb{P}(5 < X < 15)$?*

PLEASE, STOP: CONTINUE ONLY AFTER LECTURE 6.2

Problem 5. (10 points) *(Normal Approximation to the Binomial): let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Next use the Central Limit Theorem to approximate $\mathbb{P}(X = 20)$. (Hint. Notice that $\mathbb{P}(X = 20) = \mathbb{P}(19.5 < X < 20.5)$).*

Problem 6. (10 points) *The lifetime of a special type battery is a random variable with mean 40 hours and standard deviation 20 hours. Let X be a life time of a battery (A battery is used until failed!).*

- *Use Markov's inequality to estimate $\mathbb{P}(X \geq 80)$.*
- *Use Chebyshev's inequality to estimate $\mathbb{P}(30 < X < 50)$.*

Assuming a stockpile of 25 such batteries the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained. (Hint: Use Central Limit Theorem).