MATH-57091 Probability and Statistics for High-School Teachers.
Fianl Home Work, due on Wednesday December 12
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that* $(E \cup F)^c = E^c \cap F^c$.

Problem 2. *If three fair dice are tossed, what is the probability that the sum is 6? what is the probability that one of the dice shows 1 given that the sum of all three is 6?*

Problem 3. *If tree fair dice are tossed, what is the average number of the sum? What about 781 dice?*

Problem 4. *A fair coin is independently flipped* n *times,* k *times by Artem and* $n-k$ *times by Jenya. Show that the probability that Artem and Jenya flipped the same number of heads is equal to the probability that there are a total of* k *heads.*

Problem 5. *Suppose* X *is a random variable such that*

$$\mathbb{P}(X=-2)=\mathbb{P}(X=-1)=\frac{1}{6},\ \mathbb{P}(X=0)=\frac{1}{2} \text{ and}$$

$$\mathbb{P}(X=2)=\mathbb{P}(X=1)=\frac{1}{12}.$$

Please, find $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ *and* $VarX$. *Assume* Y *is an independent copy of* X*, please, find* $\mathbb{E}[3X+6Y]$ *and* $\mathbb{E}[XY]$.

Problem 6. *A random variable* X*, taking on one of the values* $0, 1, 2, \dots,$ *is said to be a Poisson r.v. with parameter* $\lambda > 0$ *if*

$$p(i) = P(X=i) = e^{-\lambda}\frac{\lambda^i}{i!},\ i = 0,1,2, \dots$$

- *Check that* $p(i)$ *really defined the probability mass function.*
- *Find* $\mathbb{E}X$.
- *Find* $Var(X)$.

Problem 7. *Assume* X *is a uniform random variable on the interval* $[-1,1]$*(i.e.* X *has a density function* $f(x)=\frac{1}{2}$*, for* $x \in [-1,1]$ *and* $f(x)=0$ *otherwise). Please, find cumulative distribution function* $F(x)=\mathbb{P}(X \le x)$, $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ *and* $VarX$.

Problem 8. *Let the probability density of* Y *be given by*

$$f_Y(x) = \begin{cases} ce^{-2x} & \text{if } x \in (0,\infty); \\ 0 & \text{otherwise.} \end{cases}$$

Please, find

- *constant c.*
- *cumulative distribution of* Y *(i.e.* $F_Y(t) = \mathbb{P}(Y < t)$*).*
- $\mathbb{P}(2 < Y)$.

Problem 9. *Suppose that we know that a number of cars produce per day by a given factory is a random number with mean* 400

- *What can be said about the probability that this week's production will be at least* 700*?*
- *If the variance of a week's production is known to equal to* 100*, then what can be said about the probability that this week's production will be between* 300 *and* 500*?*

Problem 10. *Let* $X_1, \dots, X_10$ *be independent Poisson random variable with mean* 1.

- *Use the Markov inequality to get a bound on* $P(X_1 + X_2 + ... + X_{10} \geq 15)$.
- *Use the central limit theorem to approximate* $P(X_1 + X_2 + ... + X_{10} \geq 15)$.

Problem 11. *The average life of a sample of* 10 *tires of a certain brand was* 28400 *miles. if it is known that the lifetimes of such tires are normally distributed with a standard deviation of* 3300 *miles, determine a* 95 *percent confidence interval estimate of the mean life.*

Problem 12. *To determine the average time span of a phone call made during midday, the telephone company has randomly selected a sample of* 1200 *such calls. The sample mean of these calls is* 4.7 *minutes, and the sample standard deviation is 2.2 minutes. Please, find a* 90 *percent and* 95 *confidence intervals estimate of the mean length of such calls.*

Problem 13. *A manufacturer is planing on putting out an advertisement claiming that over* x *percent of the users of his product are satisfied with it. To determine* x*, a random sample of* 500 *users was questioned. If* 92 *percent of these people indicated satisfaction and the manufacturer wants to be* 95 *percent confident about the validity of the advertisement, what value* x *should be used in the advertisement? What value should be used if the manufacturer was willing to be only* 90 *percent confident about the accuracy of the advertisement?*

Problem 14. *A farmer claims to be able to produce larger tomatoes, To test this claim, a tomato variety that has a mean diameter of size* 8.2 *centimeters with a standard deviation of* 2.4 *centimeters is used. If a sample of* 36 *tomatoes yield a sample mean* 9.1 *centimeters, does this prove that the mean size is indeed larger? (please, assume that the* *population SD remains equal to* 2.4 *and use* 5 *percent level of significance).*

Problem 15. *A car is advertised as getting at least* 31 *miles per gallon in highway driving on trips of at least* 100 *miles. Suppose the miles per gallon obtained in* 8 *independent experiments (each consisting of a nonstop highway trip of* 100 *miles) are*

$$28, 29, 31, 27, 30, 35, 25, 29$$

- *If we want to check if these data disprove the advertising claim what should we take as the null hypothesis?*
- *What we should take as an alternative hypothesis?*
- *Is the claim disproved at the* 5 *percent level of significance?*
- *What about* 1 *percent?*

Problem 16. *A high school is interested in determining whether two of its instructors are equally able to prepare students for a statewide examination in geometry. Seventy students taking geometry this semester were randomly divided into two groups of 35 each. Instructor* 1 *taught geometry to the first group, and instructor* 2 *to the second. At the end of the semester the students took the statewide examination, with the following results: Instructor* 1 - $\bar{X} = 72.6$ *and Instructor* 2 - $\bar{Y} = 74.0$. *Moreover, it is known from previous exams the grading of those instructors corresponds to the normal distribution with respective variances* $\sigma_X^2 = 6.6$ *and* $\sigma_Y^2 = 6.2$.

Can we conclude from these results that the instructors are not equally able in preparing students for the examination? Use 5 *percent level of significance.*