

MATH-57091 Probability and Statistics for High-School Teachers.
Final Home Work, due on Sunday, December 16
PLEASE, NOTE I WILL SIMPLY HAVE NO TIME TO GRADE IT
AFTER THIS
Each Problem is 6 points
Instructor: Prof. Artem Zvavitch

Problem 1. Prove that $(E \cup F)^c = E^c \cap F^c$.

Problem 2. If three fair dice are tossed, what is the probability that the sum is 6? what is the probability that one of the dice shows 1 given that the sum of all three is 6?

Problem 3. If three fair dice are tossed, what is the average number of the sum? What about 781 dice?

Problem 4. A fair coin is independently flipped n times, k times by Artem and $n - k$ times by Jenya. Show that the probability that Artem and Jenya flipped the same number of heads is equal to the probability that there are a total of k heads.

Problem 5. Suppose X is a random variable such that

$$\mathbb{P}(X = -2) = \mathbb{P}(X = -1) = \frac{1}{6}, \quad \mathbb{P}(X = 0) = \frac{1}{2} \text{ and}$$
$$\mathbb{P}(X = 2) = \mathbb{P}(X = 1) = \frac{1}{12}.$$

Please, find $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $\text{Var}X$. Assume Y is an independent copy of X , please, find $\mathbb{E}[3X + 6Y]$ and $\mathbb{E}[XY]$.

Problem 6. A random variable X , taking on one of the values $0, 1, 2, \dots$, is said to be a Poisson r.v. with parameter $\lambda > 0$ if

$$p(i) = P(X = i) = e^{-\lambda} \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

- Check that $p(i)$ really defined the probability mass function.
- Find $\mathbb{E}X$.
- Find $\text{Var}(X)$.

Problem 7. Assume X is a uniform random variable on the interval $[-1, 1]$ (i.e. X has a density function $f(x) = \frac{1}{2}$, for $x \in [-1, 1]$ and $f(x) = 0$ otherwise). Please, find cumulative distribution function $F(x) = \mathbb{P}(X \leq x)$, $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $\text{Var}X$.

Problem 8. Let the probability density of Y be given by

$$f_Y(x) = \begin{cases} ce^{-2x} & \text{if } x \in (0, \infty); \\ 0 & \text{otherwise.} \end{cases}$$

Please, find

- constant c .
- cumulative distribution of Y (i.e. $F_Y(t) = \mathbb{P}(Y < t)$).
- $\mathbb{P}(2 < Y)$.

Problem 9. Suppose that we know that a number of cars produce per day by a given factory is a random number with mean 400

- What can be said about the probability that this week's production will be at least 700?

- If the variance of a week's production is known to equal to 100, then what can be said about the probability that this week's production will be between 300 and 500?

Problem 10. Let X_1, \dots, X_{10} be independent Poisson random variable with mean 1.

- Use the Markov inequality to get a bound on $P(X_1 + X_2 + \dots + X_{10} \geq 15)$.
- Use the central limit theorem to approximate $P(X_1 + X_2 + \dots + X_{10} \geq 15)$.

Problem 11. The average life of a sample of 10 tires of a certain brand was 28400 miles. if it is known that the lifetimes of such tires are normally distributed with a standard deviation of 3300 miles, determine a 95 percent confidence interval estimate of the mean life.

Problem 12. To determine the average time span of a phone call made during midday, the telephone company has randomly selected a sample of 1200 such calls. The sample mean of these calls is 4.7 minutes, and the sample standard deviation is 2.2 minutes. Please, find a 90 percent and 95 confidence intervals estimate of the mean length of such calls.

Problem 13. A manufacturer is planing on putting out an advertisement claiming that over x percent of the users of his product are satisfied with it. To determine x , a random sample of 500 users was questioned. If 92 percent of these people indicated satisfaction and the manufacturer wants to be 95 percent confident about the validity of the advertisement, what value x should be used in the advertisement? What value should be used if the manufacturer was willing to be only 90 percent confident about the accuracy of the advertisement?

Problem 14. A farmer claims to be able to produce larger tomatoes, To test this claim, a tomato variety that has a mean diameter of size 8.2 centimeters with a standard deviation of 2.4 centimeters is used. If a sample of 36 tomatoes yield a sample mean 9.1 centimeters, does this prove that the mean size is indeed larger? (please, assume that the population SD remains equal to 2.4 and use 5 percent level of significance).

Problem 15. A car is advertised as getting at least 31 miles per gallon in highway driving on trips of at least 100 miles. Suppose the miles per gallon obtained in 8 independent experiments (each consisting of a nonstop highway trip of 100 miles) are

28, 29, 31, 27, 30, 35, 25, 29

- If we want to check if these data disprove the advertising claim what should we take as the null hypothesis?
- What we should take as an alternative hypothesis?
- Is the claim disproved at the 5 percent level of significance?
- What about 1 percent?

Problem 16. A high school is interested in determining whether two of its instructors are equally able to prepare students for a statewide examination in geometry. Seventy students taking geometry this semester were randomly divided into two groups of 35 each. Instructor 1 taught geometry to the first group, and instructor 2 to the second. At the end of the semester the students took the statewide examination, with the following results: Instructor 1 - $\bar{X} = 72.6$ and Instructor 2 -

$\bar{Y} = 74.0$. Moreover, it is known from previous exams the grading of those instructors corresponds to the normal distribution with respective variances $\sigma_X^2 = 6.6$ and $\sigma_Y^2 = 6.2$.

Can we conclude from these results that the instructors are not equally able in preparing students for the examination? Use 5 percent level of significance.

Problem 17. The following are average 2003 math SAT scores in a sample of states, along the percentage of graduating seniors who took the test

State	AverageScore	Percentage
Arizona	525	38
California	519	54
Indiana	504	63
Missouri	583	8
Louisiana	559	8
Oregon	527	57
Virginia	510	71
Wisconsin	594	7
Texas	491	57
Vermont	512	70

Please, find the estimated regression line.

Problem 18. The following data set presents the heights of 12 male law school classmates whose law school examination scores were roughly equal. It also gives their annual salaries 5 years after graduation. Each went into corporate law. The height is in inches, and the salary is in units of \$1000.

Height	Salary
64	111
65	114
66	108
67	123
69	97
70	116
72	125
72	108
74	142
74	122
75	110
76	134

Do the given data establish the hypothesis that a lawyer's salary is related to his height? Please, use the 5 percent level of significance.