TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 1, due on THURSDAY SEPTEMBER 4,
Instructor: Prof. Artem Zvavitch

Problem 1. *If tree fair dice are tossed, what is the probability that the sum is 6? what is the probability that one of the dice shows 1 given that the sum of all three is 6?*

Problem 2. *If tree fair dice are tossed, what is the average number of the sum? What about 781 dice?*

Problem 3. *Suppose X is a random variable such that*

$$\mathbb{P}(X=-2)=\mathbb{P}(X=-1)=\frac{1}{6},\ \mathbb{P}(X=0)=\frac{1}{2}\ \text{and}\ \mathbb{P}(X=2)=\mathbb{P}(X=1)=\frac{1}{12}.$$

Please, find $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $VarX=\mathbb{E}[X-\mathbb{E}X]^2$. Assume Y is an independent copy of X, please, find $\mathbb{E}[3X+6Y]$ and $\mathbb{E}[XY]$. Please, also compute $\mathbb{E}[X|X+Y=0]$.

Problem 4. *Assume X is a uniform random variable on the interval $[-1,1]$ (i.e. X has a density function $f(x)=\frac{1}{2}$, for $x\in[-1,1]$ and $f(x)=0$ otherwise). Please, find cumulative distribution function $F(x)=\mathbb{P}(X\leq x)$, $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $VarX$.*

Problem 5. *The join density of random variables X and Y is given by*

$$f(x,y)=\frac{e^{-x/y}e^{-y}}{y},\ \text{where}\ 0<x<\infty, 0<y<\infty.$$

Compute $E[X|Y=y]$.

Problem 6. *Let X_i, $i\geq 0$ be independent and identically distributed random variable with probability math function $p(k)=\mathbb{P}(X_i=k)$, for $k=1,\cdots,m$ (and $\sum_{j=1}^{m}p(j)=1$).*
Find $\mathbb{E}[N]$, where $N=\min\{n: X_n=X_0\}$. (Hint: use conditional expectation!)

Problem 7. (5 bonus points) *Assume X and Y are a uniform random variables on the interval $[-1,1]$. Find $\mathbb{E}[X|X+Y=y]$.*