TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 2, due on THURSDAY SEPTEMBER 11,
Instructor: Prof. Artem Zvavitch

Problem 1. *Assume X is a uniform random variable on the interval $(0,1)$, find* $\mathbb{E}[X^2|X < \frac{1}{2}]$.

Problem 2. *The join density of random variables X and Y is given by*

$$f(x,y) = \frac{e^{-y}}{y}, \text{ where } 0 < x < y, 0 < y < \infty.$$

Compute $E[X^2|Y = y]$.

Problem 3. *If* $\mathbb{E}[X|Y = y] =$ *constant for all* y, *show that* $Cov(X,Y) = 0$. *(Where covariance of two random variables X and Y is defined by* $Cov(X,Y) = \mathbb{E}([X - \mathbb{E}X][Y - \mathbb{E}Y])$.*)*

Problem 4. *Suppose that whether of not it rains today depends on previous weather conditions through the last three days. Show how this system may be analyzed by using Markov chain. How many states are needed?*

Problem 5. *Suppose that coin* **1** *has probability* 0.7 *of coming up heads, and coin* **2** *has probability* 0.6 *of coming up heads. If the coin flipped today comes up heads, then we select coin* **1** *to flip tomorrow, and if it comes up tails, then we select coin* **2** *to flip tomorrow. If the coin initially flipped is equally likely to be coin* **1** *or* **2**, *then what is the probability that the coin flipped on the third day after the initial flip is coin* **1**.

Problem 6. *Specify the classes of the following Markov chain and determine whether they are transient or recurrent:*

$$\mathbf{P} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$